Z3 for iProver-Eq: Efficient Ground Solving for Instantiation-based First-order Reasoning

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Joint work with Konstantin Korovin

Z3 Special Interest Group Meeting
3rd November 2011

Motivation

The main application of automated reasoning is verification of software, hardware, protocols etc.

Reasoning should

- scale to industrial-size problems and
- provide succinct formalisation.

First-order Logic

- High expressivity: quantifiers $\forall$, $\exists$
- Decidable fragments
- Resolution/superposition
- Weak ground reasoning and modulo theories

SAT / Quantifier-free SMT

- High efficiency
- Modulo theories
- DPLL/congruence closure
- Weak quantifier reasoning

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Instantiation-based Methods: The Idea

Is a given closed formula $\forall \bar{x}\ \varphi(\bar{x})$ a theorem?

A refutationally complete method:

1. Guess finite number of ground instances of $\varphi(\bar{x})$
2. Test ground satisfiability

Benefits:

- Keep expressivity of first-order logic
- Exploit efficiency of SAT and *QF_SMT*

Instantiation-based Methods: The Main Problem

Core question in instantiation-based reasoning

How do we find the ground instances to witness first-order unsatisfiability?

- Decidable if there are finitely many ground instances
- Harder the "more" ground instances there are
- Differences between calculi:
  - generation of instances
  - integration of propositional solving

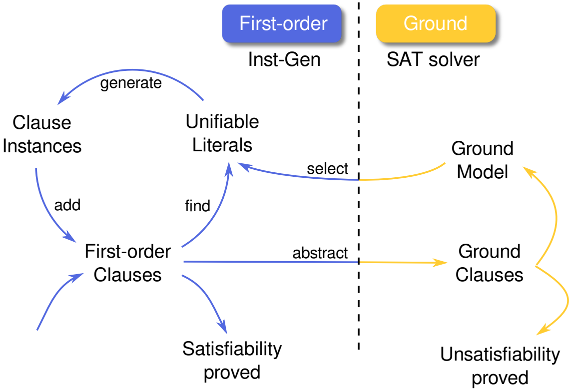
Features of Inst-Gen

- Modular combination of first-order and ground reasoning
- Ground reasoning delegated to off-the-shelf solver
- Very efficient for the EPR fragment
- Applied for hardware verification with bounded model checking (Intel)

- Non-equational variant related to Resolution
- Superposition-style equational reasoning
- Theory reasoning possible

- Implemented in *iProver* and *iProver-Eq*
- *[Korovin & Sticksel IJCAR 2010]* and *[Korovin & Sticksel LPAR 2010]*

The Inst-Gen Method



Inst-Gen: Ground Abstraction and Selection

First-order clauses

$$\neg Q(f(x))$$

$$\neg P(f(f(y)))$$

$$P(f(z)) \lor Q(z)$$

Ground abstraction with $\perp$

$$\neg Q(f(\perp))$$

$$\neg P(f(f(\perp)))$$

$$P(f(\perp)) \lor Q(\perp)$$

- Select literals which are true in ground abstraction

Fail to extend ground model to first-order

$$\neg P(f(f(y))) \models \neg P(f(f(a)))$$

$$P(f(z)) \models P(f(f(a)))$$

- Model has to be refined on the conflict

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Inst-Gen: Instance Generation Inference

Inst-Gen Inference

$$\frac{\neg P(f(f(y))) \qquad P(f(z)) \vee Q(z)}{\neg P(f(f(y))) \qquad P(f(f(y))) \vee Q(f(y))}\,[z \to f(y)]$$

- Inference with most general unifier on $\neg P(f(f(y)))$ and $P(f(z))$ which are are selected and complementary.

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Inst-Gen Modulo Equality

- Inst-Gen inference rule not sufficient
- Obvious step from Resolution to Superposition to generate instances is incomplete
- Set of literals of any size can be contradictory

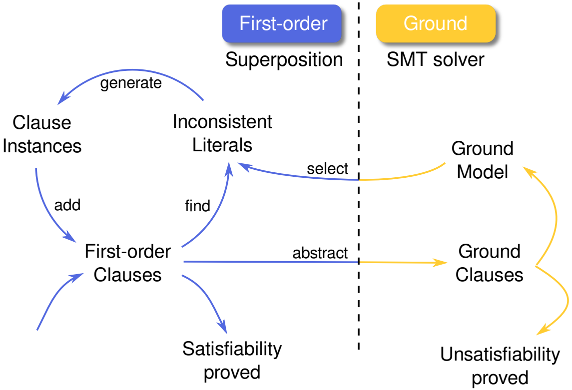
$$\{\, f(x) \not\simeq f(a) \,\}$$

$$\{\, f(x) \simeq a, \quad f(a) \not\simeq a \,\}$$

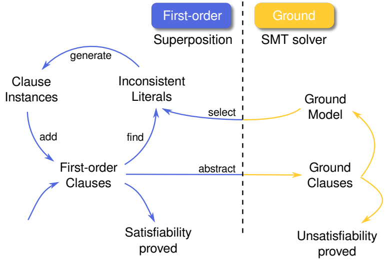
$$\{\, h(y) \simeq y, \quad f(h(x)) \simeq c, \quad f(a) \not\simeq c \,\}$$

- Labelled unit superposition calculus
- Instance generation from labels of contradictions
- Ground solver modulo equality (*QF_UF* solver)

The Inst-Gen-Eq Method



Efficient Ground Solving in Inst-Gen



Cooperation with SMT solver

- Incrementally add clauses
- Test unsatisfiability
- Query truth value of literals

Beyond the basics

- Global propositional subsumption
- Minimise changes to selection
- Auxiliary “soft” assertions

Statistics and experimental results

Global Propositional Subsumption

Generalise grounding by $\perp$ to a set of constants Σ_c, consider substitutions $\gamma \in \Omega$, e.g. $[x \to c_1, y \to c_2, \dots]$

Propositional Simplification

$$\frac{D \vee D'}{D'} \qquad \text{if } C_1\gamma_1, \dots C_k\gamma_k \models D\gamma$$

- Finding a minimal D' is linear in length of $D \vee D'$
- Individual ground constant for each variable
- Separate instance of Z3
- Approximation is sufficient:
  - consider only one $\gamma \in \Omega$ and
  - limit runtime of solver

Z3 Models and iProver Selection

- Semantic selection: in each clause one literal L such that $L\bot$ is true in the ground model
- Saturation process: Changing selection removes L and enters L', inferences with clause to be repeated

Tweak model to preserve selection

Local Is there is a model such that the previous selection for this clause can be kept?

Global Which model requires the least changes across all clauses to the current selection?

Inst-Gen(-Eq) calculus is complete for any model, hence approximate answers suffice

Soft Contraints and Unsatisfiable Cores

- Auxiliary literals for tracking purposes

  Proofs: Input clauses in unsatisfiable core

  Answers: Transform

  $$\exists x \ \varphi(x)$$

  to

  $$\exists x \ \varphi(x) \wedge answer(x)$$

- Soft constraints and unsat cores for incremental solving

  Bounded model checking: Enumerate states, transfer information from one bound to next

  Finite model finding: Enumerate domain constants

Two Incarnations of Z3

Satisfiability solver

- Witness unsatisfiability and select literals
- Full solving with model
- Grounding with $\perp$
- Tweak model to preserve previous selections (soft constraints, unsat cores)

Simplification solver

- Global propositional subsumption
- Fast and incomplete
- Unit propagation, bound number of decisions
- Grounding with $\perp$ and $[x \to c_1, y \to c_2, \ldots]$

Specifics of Ground Reasoning in iProver-Eq

- ground problems are typically simple $< 1s$
- frequent solver calls typically > 1000
- Incrementality: clauses are added inrementally, hundreds of thousands in some applications

Experimental Results

iProver-Eq with CVC3 vs. Z3 on
TPTP v5.2.0 problems with equations only (9,507 total)

Number of problems

	solved	only	faster	by 50%	by 100%
CVC3	2,468	87	663	57	30
Z3	2,510	129	1,718	551	317

- Ground model strongly influences first-order reasoning
- Problems for solvers are structurally simple
- Most effort on adaption of selection to model

iProver-Eq and Z3

Features used:

- Incrementally assert clauses
- Push and pop to find different model
- Check satisfiability, also with assumptions
- Evaluate literals in calculated model

Wishlist/To-Do:

- Access to learnt clauses
- Default decision value for literals
- Soft enforcing of truth values
- Fast solving for simplifications
- Use tactics to our advantage

Summary

Instantiation-based reasoning à la Inst-Gen and Inst-Gen-Eq

- Sound and complete first-order method
- Modulo equality and modulo theories
- SMT solver for ground reasoning

Future Work

- Improve efficiency of cooperation with solver
- Tune to applications
- First-order reasoning modulo theories