Labelled Unit Superposition Calculi for Instantiation-Based Reasoning

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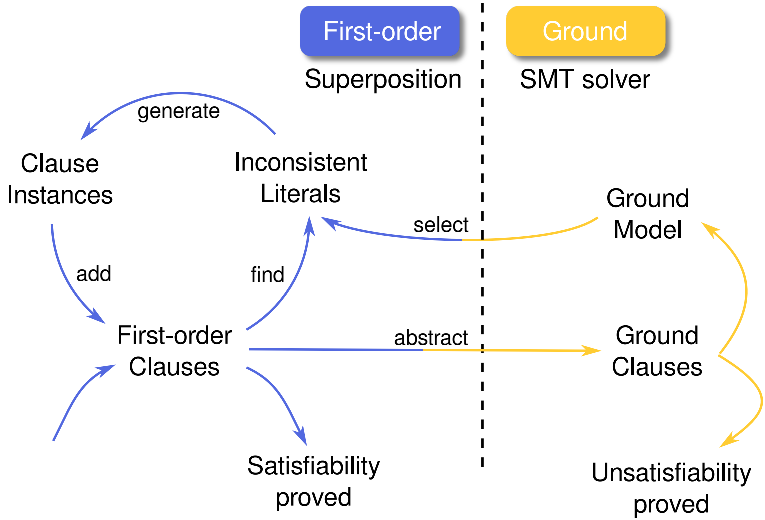
Background

- Instantiation-based methods for first-order logic
  - Decision procedure for Bernays-Schönfinkel fragment (verification, planning/scheduling, knowledge representation)
  - Good performance in plain first-order logic
  - Complementary to “traditional” first-order calculi
- Equational reasoning
  - Essential part in theory reasoning
  - Natural concept in many applications
  - Not well explored in instantiation-based setting
- Here: Instantiation-based calculus Inst-Gen-Eq
  - Ganzinger and Korovin [2004]
  - Complete for first-order clause logic modulo equality

The Inst-Gen-Eq Method



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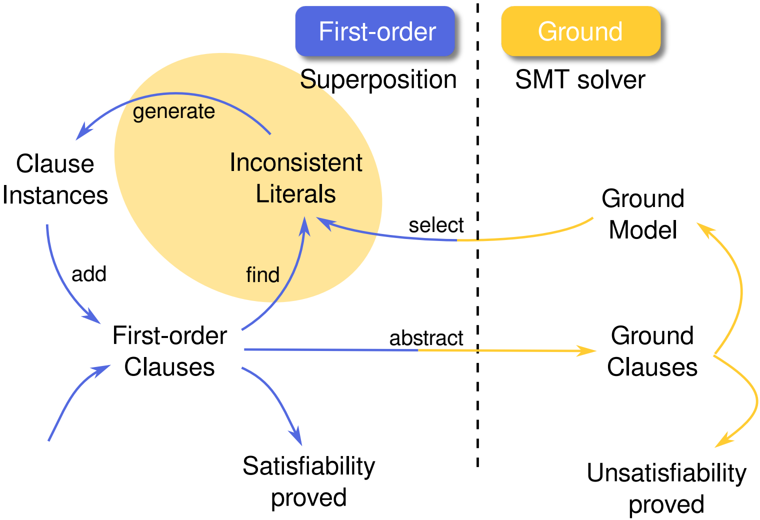
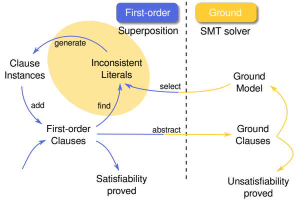


Finding Inconsistencies and Generating Instances



Key issues

- Unit reasoning to find inconsistent literals
- Generating clause instances from superposition proofs
- All (non-redundant) proofs to be considered

Our approach

- Labelled Unit Superposition
- Different label structures
  - Sets
  - AND/OR trees
  - OBDDs

Inst-Gen-Eq: Finding Inconsistencies

First-order clauses

$$f(x,y) \simeq f(y,x)$$

$$f(u,v) \not\simeq g(z) \ \lor \ u \simeq z$$

$$f(a,b) \simeq g(c)$$

$$a \not\simeq b$$

Ground abstraction with $\perp$

$$f(\perp,\perp) \simeq f(\perp,\perp)$$

$$f(\perp,\perp) \not\simeq g(\perp) \ \lor \ \perp \simeq \perp$$

$$f(a,b) \simeq g(c)$$

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Unit superposition proof: Selected literals inconsistent

$$\dfrac{\dfrac{\dfrac{f(a,b) \simeq g(c) \qquad \dfrac{f(x,y) \simeq f(y,x) \qquad f(u,v) \not\simeq g(z)}{f(v,u) \not\simeq g(z)}\,[u/x, v/y]}{g(c) \not\simeq g(z)}\,[a/v, b/u]}{\square}\,[c/z]}{}$$

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Inst-Gen-Eq: Generating Instances

Unit superposition proof: Substitution extraction

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Instances from Each Inconsistency Required

Proof of inconsistency (1)

$$\cfrac{f(a,b) \simeq g(c) \qquad \cfrac{f(x,y) \simeq f(y,x) \qquad f(u,v) \not\simeq g(z)}{f(v,u) \not\simeq g(z)}\,[u/x, v/y]}{\cfrac{g(c) \not\simeq g(z)}{\square}\,[c/z]}\,[a/v, b/u]$$

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The Labelling Approach

- Distinguish literal variants by labels
- Explicit *merging* inference to combine variants

Components

- *Closure* $C \cdot \theta$: clause C and substitution θ
- Initially $\{C \cdot []\} : L$ where L is selected in C
- Label of contradiction $\Box$ contains instances to be added

Advantages

- Eager extraction of instances after each inference in label
- Cycle-free proofs with sharing of subproofs
- Uniform treatment of literal variants
- Preserve proof structure for redundancy elimination

Inference Rules in Labelled Unit Superposition

Superposition

$$\frac{\mathcal{T} : l \simeq r \qquad \mathcal{T}' : L[l']}{(\mathcal{T} \sqcap \mathcal{T}')\sigma : L[r]\sigma}\ (\sigma) \qquad \sigma \text{ is mgu of } l \text{ and } l'$$

Variant merging

$$\frac{\mathcal{T} : L \qquad \mathcal{T}' : L'}{\mathcal{T} \sqcup \mathcal{T}'\sigma : L}\ (\sigma) \qquad L = L'\sigma,\ \sigma \text{ is a renaming}$$

Equality resolution

$$\frac{\mathcal{T} : (l \not\simeq r)}{\mathcal{T}\sigma : \square}\ (\sigma) \qquad \sigma \text{ is mgu of } l \text{ and } r$$

- No labels in side conditions
- Label $\mathcal{T}$ is either a set, AND/OR tree or OBDD
- $\sqcap$ and $\sqcup$ dependant on implementation of labels

Set Labelled Unit Superposition

- Label is a set of closures
- Set union $\cup$ in both merging $\sqcup$ and superposition $\sqcap$

Superposition

$$\frac{\{C \cdot []\} : f(x,y) \simeq f(y,x) \quad \{D \cdot []\} : f(u,v) \not\simeq g(z)}{\{C \cdot [u/x, v/y], D \cdot []\} : f(v,u) \not\simeq g(z)} \; [u/x, v/y]$$

Merging $f(u,v) \not\simeq g(z)$ and $f(v,u) \not\simeq g(z)$ with $[u/v, v/u]$

$$\{D \cdot [], C \cdot [v/x, u/y], D \cdot [v/x, u/y]\} : f(u,v) \not\simeq g(z)$$

Label of the contradiction $\square$

$$\{D \cdot [a/u, b/v, c/z], E \cdot [], C \cdot [b/x, a/y], D \cdot [b/u, a/v, c/z]\}$$

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- Preserve Boolean structure of proofs
- Closure is a propositional variable in an AND/OR tree
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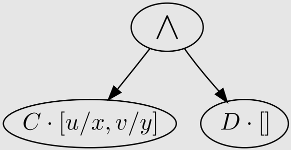
Label of $f(v, u) \not\approx g(z)$ after superposition



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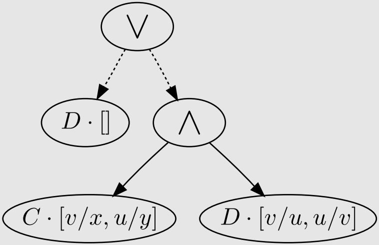
Label of $f(v, u) \not\simeq g(z)$ with variants merged



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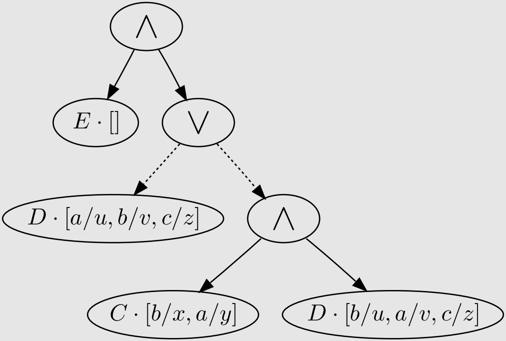
Label of the Contradiction $\square$



OBDD Labelled Unit Superposition

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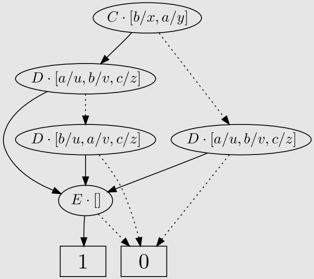


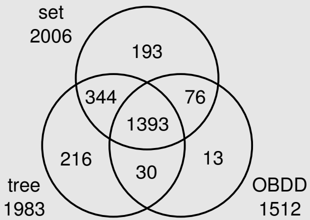
Disadvantages of trees

- Not produced in normal form
- Sequence of inferences determines shape
- Potential growth *ad infinitum*

- OBDD as normal form
- Maintenance effort
- Reordering required

Sets vs. Trees vs. OBDDs

Number of solved problems



- TPTP v4.0.1
- Equational problems only

Set

- Normal form
- Approximate redundancy elimination

AND/OR tree

- No normal form
- Precise redundancy elimination

OBDD

- Normal form
- Precise redundancy elimination

Summary and Further Work

- Labelled calculus for elegant and efficient unit reasoning
  - Literal variants
  - Sharing of proofs
  - Redundancy elimination
- Sound and complete calculus
- Three different labelling structures

- Implementation in *iProver-Eq*
- Evaluation on TPTP v4.0.1

- Hybrid label structures
- Other uses of information from labels